JBA-003-1163004 Seat No. ______

M. Sc. (Sem. III) Examination

December - 2019

EMT-3011 : Mathematics

(Differential Geometry)

Faculty Code : 003
Subject Code : 1163004

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are 5 questions.
(2) Attempt all the questions.
(3) Each question carries equal marks.

1 Attempt any **seven** : **14**

(1) Define : Regular curve and regular curve segment.

(2) Define : Proper co-ordinate patch.

(3) Is the curve a $\alpha(t) = \left(t^3, t^2, 100t\right)$ regular ? Justify your answer.

(4) Define : Arc length.

(5) Define : Unit speed curve.

(6) Define : The tangent space and the normal space.

(7) Define : Normal curvature and Geodesic curvature.

(8) Define: Simple surface.

(9) Define : Tangent vector to a simple surface.

(10) Define : Tangent vector field.

2 Attempt the following : **14**

(a) Define right circular helix and find the arc length of the helix $\alpha(t) = \left(a\cos t, a\sin t, at\tan\alpha\right)$.

(b) Define : Reparametrization. If $g:[c,d]\rightarrow[a,b]$ is a reparametrization of a curve segment $\alpha:[a,b]\rightarrow R^3$ then show that the length of α is equal to the length of $\beta=\alpha\circ g$.

OR

(b) Reparametrize the curve $\alpha(t)=(r\cos t, r\sin t, 0)$ by its arc length and also find its curvature (where $r>0$).

3 Attempt the following : **14**

(a) For the circular helix $\alpha(t)=(r\cos\omega s, r\sin\omega s, h\omega s)$, compute Frenet - Serret appartus.

$\left(\text{where } \omega=\left(r^2+h^2\right)^{-\frac{1}{2}}\right)$.

OR

(a) Show that $\alpha(s)=\left(\frac{(1+s)^{\frac{3}{2}}}{3},\frac{(1-s)^{\frac{3}{2}}}{3},\frac{s}{\sqrt{2}}\right)$ is a unit speed curve and compute its Frenet Serret appartus.

(b) Show that the curve $\alpha(S)=\left(\frac{5}{13}\cos S,\frac{8}{13}-\sin S,-\frac{12}{13}\cos S\right)$ is a unit speed curve. Also compute its Frenet – Serret appartus.

4 Attempt the following : **14**

(a) Prove that : The set of all tangent vectors to a simple surface $x:u\rightarrow R^3$ at P is a vector space.

(b) State and prove Frenet - Serret theorem.

5 Attempt any **two** : **14**

(a) If $x : u \to R^3$ is a simple surface and $f : v \to u$ is a co-ordinate transformation such that $y = x \circ f$ then prove that

(i) The tangent plane to the simple surface x at $P = x\left(f\left(a, b\right)\right)$ is equal to the tangent plane to the simple surface y at $P = y\left(a, b\right)$.

(ii) The normal to the surface x at P is same as the normal to the surface y at P except possibly it may have the opposite sign.

(b) Let α(s) be a unit speed curve whose image lies on a sphere of radius r and centre m then show that $k \neq 0$. Also if $r \neq 0$ then $\alpha - m = \rho N - \rho' \sigma \beta$ and

$r^2 = \rho^2 + \left(\rho' \sigma\right)^2$ (where $\rho = \frac{1}{k}$ and $\sigma = \frac{1}{\tau}$).

(c) Find the co-efficient of second fundamental form and Christoffel symbols for the surface

$x\left(u^1, u^2\right) = \left(u^1, u^2, f\left(u^1, u^2\right)\right)$.

(d) Find the curvature of the curves

(i) $2x - 3y + 5 = 0$

(ii) $x^2 + y^2 + 6x - 8y + 64 = 0$